Quiz 4, Calculus I

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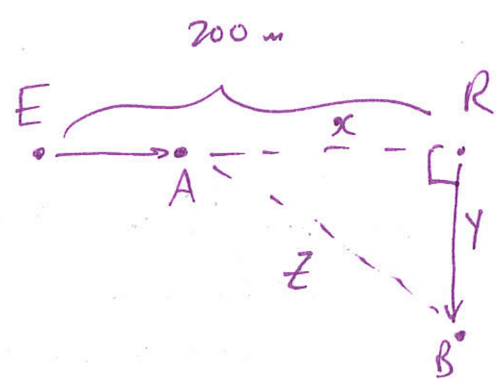
10:43

Name: Key

1. (4 points) The towns of Evatopolis and Ronanville are 200 miles apart, connected by a straight road that runs east to west. Car A leaves from Evatopolis heading toward Ronanville at noon. Also at noon, Car B leaves Ronanville heading directly south. If Car A is driving 40 mph and Car B is driving 65 mph, *use calculus* to find out how fast the (straight line) distance between the two cars is changing at exactly 2pm. Round your answer to the nearest 0.01 mph.



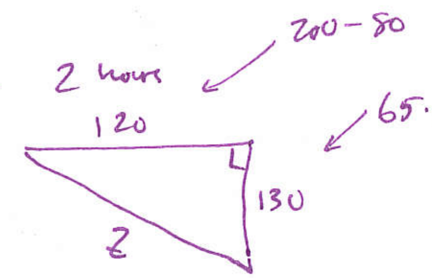
$\frac{dx}{dt} = -40$

$\frac{dy}{dt} = 65$

WTF: $\left.\frac{dz}{dt}\right|_{t=2\text{ hours}}$

$\frac{d}{dt}\left(x^2+y^2 = z^2 \right)$

at 2 hours



$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$\Rightarrow 2(120)(-40) + 2(130)65 = 2\left(\sqrt{120^2+130^2}\right)\frac{dz}{dt}$

$z^2 = 120^2 + 130^2$

$\Rightarrow z = \sqrt{120^2+130^2}$

$\frac{dz}{dt} = \frac{240(-40)+260(65)}{2\sqrt{120^2+130^2}} = \boxed{20.63 \text{ mph}}$

2. (3 points) (a) Calculate the linearization at $p = 0$ for both $f(x) = \sin x$ and $g(x) = \tan x$, and confirm that for each function you get $L(x) = x$.

(b) For $x = \frac{\pi}{6}$, does the linearization above give a better approximation for $\sin x$ or for $\tan x$? Explain your reasoning (you will need to use a calculator or a graph to assist your explanation, most likely).

for $\sin = f(x)$: $L(x) = f(p) + f'(p)(x-p)$

$L(x) = \sin 0 + f'(0)(x-0)$

$L(x) = 0 + 1(x) = x$ ✓

for $g(x) = \tan x$: $L(x) = \tan 0 + 1(x-0)$

$L(x) = 0 + x = x$ ✓

for $\sin x$

$f'(x) = \cos x$

for $\tan x$

$g'(x) = \sec^2 x$

$\Rightarrow f'(0) = \cos 0 = 1$

$g'(0) = \sec^2 0 = \frac{1}{1} = 1$

(b) $L\left(\frac{\pi}{6}\right) = \frac{\pi}{6} \approx 0.523599$

$\sin\left(\frac{\pi}{6}\right) = \frac{1}{2} = 0.5$

$\tan\left(\frac{\pi}{6}\right) = \frac{1}{\sqrt{3}} \approx 0.57735$

so $\boxed{\sin\left(\frac{\pi}{6}\right) \text{ is closer to } L(x)}$ b/c

difference is 0.0235

other is 0.053

3. (3 points) Find the absolute maximum and minimum for $y = 5e^{x^3-4x}$ on the interval [-2,~~2~~] 2.5

$y' = 5e^{x^3-4x} \cdot (3x^2-4)$

$0 = 5e^{x^3-4x}(3x^2-4)$

$e^{x^3-4x} = 0$

dne

$3x^2 - 4 = 0$

$\Rightarrow x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

$x \approx \pm 1.1547$

$f(-2) = 5$

$f(-1.1547) = 108.71$

$\boxed{f(1.1547) = 0.22998}$

abs. min

$f(2.5) = 1386.4$

abs max